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Sum Of Power N Divisor Cordial Labeling Of Herschel Graph

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Abstract:
A sum of Power n divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \rightarrow \{1,2,3,\dots,|V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $(f(u)+f(v))^n$ and 0 otherwise. The number of edges labeled with 0 and the number of edges labeled with 1 differ atmost 1. A graph with a sum of power n divisor cordial labeling is called a sum of power n divisor cordial graph. We establish in this paper that Herschel graph H_s, fusion of any two adjacent vertices of degree 3 in Herschel graph H_s, duplication of any vertex of degree 3 in a Herschel graph H_s, switching of a central vertex in the Herschel graph H_s, path union of two copies of Herschel graph H_s are sum of power n divisor cordial graphs.

Keywords: Divisor cordial labeling, Sum of power n divisor cordial labeling, fusion, duplication, switching, path union.

Introduction:
By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory we refer to Harary [3]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Preetha lal and M. Jaslin Melbha [6,7,8] introduced the concept of sum of power n divisor cordial labeling. Varatharajan et al.[13] introduced the concept of divisor cordial labeling. Vaidya and Shah [12] proved that some star and bistar related graphs are divisor cordial labeling. Rokad and Godasara [9] have discussed the Fibonacci cordial labeling of some special graphs. For dynamic survey of various graph labeling we refer to Gallian [1]. Lourdusamy and Patrick [5] introduced the concept of sum divisor cordial labeling. Our primary objective of this paper is to prove the Herschel

graph and some graph operations in Herschel graph namely, fusion of any two adjacent vertices of degree 3 in Herschel graph H_s, duplication of any vertex of degree 3 in a Herschel graph H_s, switching of a central vertex in the Herschel graph H_s, path union of two copies of Herschel graph H_s are sum of power n divisor cordial graphs.

1. Preliminaries

Definition 1.1. A sum of Power n divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1,2,3,\dots,|V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $(f(u)+f(v))^n$ and 0 otherwise. The number of edges labeled with 0 and the number of edges labeled with 1 differ atmost 1. A graph with a sum of power n divisor cordial labeling is called a **sum of power n divisor cordial graph**.

Definition 1.2. A **Herschel graph H_s** is a bipartite graph with 11 vertices and 18 edges.

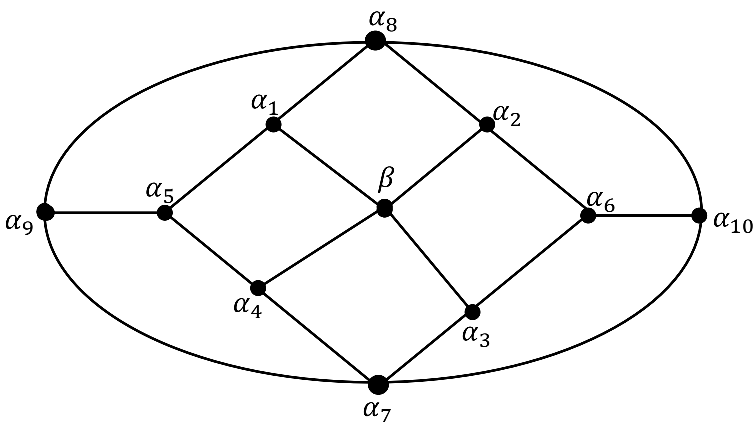

Figure 1. Herschel graph H_s

In this paper, we always fix the position of vertices $\beta, \alpha_1, \alpha_2, \dots, \alpha_{10}$ of H_s as indicated in the above figure 1, unless or otherwise specified.

Definition 1.3. Let α and β be two distinct vertices of a graph G. A new graph G_1 is constructed by **fusing** (identifying) two vertices α and β by a single vertex γ in G_1 such that every edge which was incident with either α or β in G now incident with γ in G_1.

Definition 1.4. Duplication of a vertex β_k of a graph G produces a new graph G_1 by adding a vertex β'_k with $N(\beta_k) = N(\beta'_k)$. In other words, a vertex β'_k is said to be a duplication of β_k if all the vertices which are adjacent to β_k are now adjacent to β'_k.

Definition 1.5. A **Vertex switching** G_1 is obtained by taking a vertex β of G, removing the entire edges incident with β and adding edges joining β to every vertex which are non-adjacent to β in G.

Definition 1.6. Let G be a graph and let $G_1 = G_2 = \cdots = G_n = G$, where $n \geq 2$. Then the graph obtained by adding an edge from each G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called the **path union** of G.

2. Main Results

Theorem 2.1. The Herschel graph H_s is a sum of power n divisor cordial graph.

Proof. Let $G = H_s$ be Herschel graph and let β be the central vertex and α_i $(1 \leq i \leq 10)$ be the remaining vertices of the Herschel graph. Then $|V(G)| = 11$ and $|E(G)| = 18$. Define the vertex labeling $f: V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ as follows;

$f(\beta) = 11$, $f(\alpha_2) = 1$, $f(\alpha_8) = 4$,

$f(\alpha_i) = i, i = 3, 9, 10,$

$f(\alpha_i) = i + 1, i = 1, 4, 5, 6, 7.$

From the above labeling pattern, we have $e_f(0) = e_f(1) = 9$.

Hence $|e_f(0) - e_f(1)| \leq 1$. Thus G is sum of power n divisor cordial graph.

Example 2.2. A sum of power n divisor cordial labeling of Herschel graph H_s is given below.

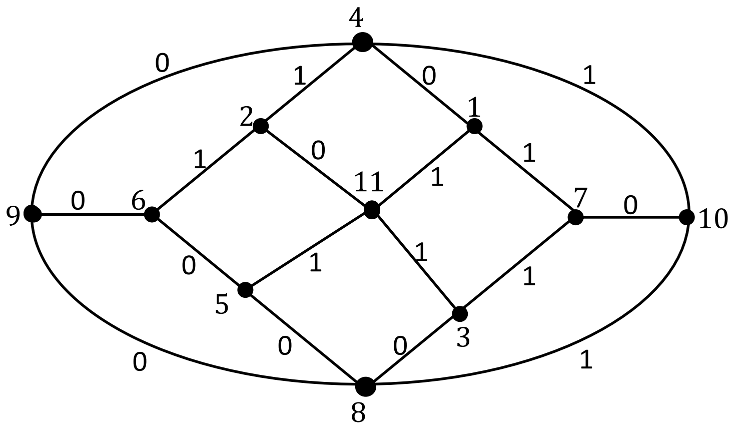

Figure 2.1.

Theorem 2.3. The fusion of any two adjacent vertices of degree 3 in the Herschel graph H_s is a sum of power n divisor cordial graph.

Proof. Let $G = H_s$ be Herschel graph with $|V(H_s)| = 11$ and $|E(H_s)| = 18$. Let β be the central vertex of the Herschel graph and it has 3 vertices of degree 4 and 8 vertices of degree 3. Let G be the graph obtained by fusion of any two adjacent vertices of degree 3 in the Herschel graph H_s. Then $|V(G)| = 10$ and $|E(G)| = 10$. Define the vertex labeling $f: V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ as follows;

Case (i). Fusion of α_6 and α_{10}.

Suppose that α_6 and α_{10} are fused together as a single vertex α.

$f(\beta) = 1$, $f(\alpha_3) = 7$,

$f(\alpha) = 4$,

$f(\alpha_i) = i + 1, i = 1, 2, 4, 5, 7, 8, 9.$

Case (ii). Fusion of α_6 and α_2.

Suppose that α_6 and α_2 are fused together as a single vertex α.

$f(\beta) = 1$, $f(\alpha_4) = 7$, $f(\alpha_{10}) = 4$,

$f(\alpha) = 2$,

$f(\alpha_i) = i + 1, i = 5, 7, 8, 9,$

$f(\alpha_i) = i + 2, i = 1, 3.$

Case (iii). Fusion of α_6 and α_3.
Suppose that α_6 and α_3 are fused together as a single vertex α.
$f(\beta) = 1$, $f(\alpha_1) = 9$, $f(\alpha_{10}) = 4$,
$f(\alpha) = 2$,
$f(\alpha_i) = i, i = 7, 8,$
$f(\alpha_i) = i + 1, i = 2, 4, 5, 9.$
Case (iv). Fusion of α_5 and α_9.
Suppose that α_5 and α_9 are fused together as a single vertex α.
$f(\beta) = 1, f(\alpha_3) = 7,$
$f(\alpha) = 4,$
$f(\alpha_i) = i, i = 6, 10,$
$f(\alpha_i) = i + 1, i = 1, 2, 4, 7, 8.$
Case (v). Fusion of α_5 and α_4.
Suppose that α_5 and α_4 are fused together as a single vertex α.
$f(\beta) = 1, f(\alpha_3) = 7, f(\alpha_9) = 5,$
$f(\alpha) = 4,$
$f(\alpha_i) = i, i = 6, 10,$
$f(\alpha_i) = i + 1, i = 1, 2, 7, 8.$
Case (vi). Fusion of α_5 and α_1.
Suppose that α_5 and α_1 are fused together as a single vertex α.
$f(\beta) = 1, f(\alpha_3) = 7, f(\alpha_9) = 4,$
$f(\alpha) = 2,$
$f(\alpha_i) = i, i = 6, 10,$
$f(\alpha_i) = i + 1, i = 2, 4, 7, 8.$
From all the above cases, $|e_f(0) - e_f(1)| \leq 1$. Thus G is sum of power n divisor cordial graph.

Example 2.4. A sum of power n divisor cordial labeling of fusion α_6 and α_{10} in H_s is given below.

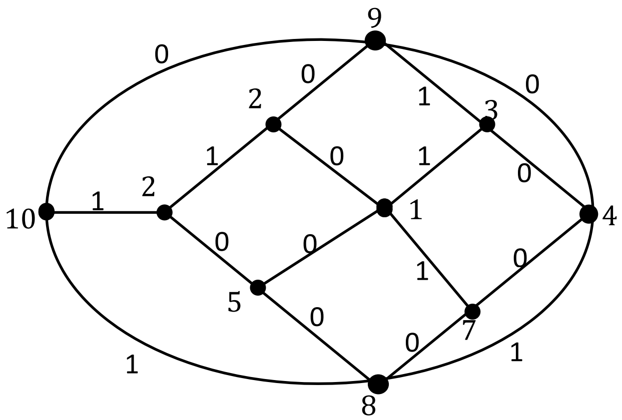

Figure 2.2.

Theorem 2.5. The duplication of any vertex of degree 3 in a Herschel graph H_s is a sum of power n divisor cordial graph.

Proof. Let H_s be Herschel graph with $|V(H_s)| = 11$ and $|E(H_s)| = 18$. Let β be the central vertex and α'_k be the duplication of the vertex α_k in the Herschel graph H_s.Let G be the graph obtained by duplicating the vertex α_k of degree 3 in the Herschel graph H_s. Then $|V(G)| = 12$ and $|E(G)| = 21$. Define the vertex labeling $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ as follows;

Case (i). Duplication of vertex α_k, where $k = 1, 2, 3, 4, 5, 6, 10$.

$f(\beta) = 1$, $f(\alpha_3) = 11$, $f(\alpha_8) = 4$, $f(\alpha'_k) = 12$

$f(\alpha_i) = i, i = 9,10,$

$f(\alpha_i) = i + 1, i = 1, 2, 4, 5, 6, 7.$

Case (ii). Duplication of vertex α_9.

$f(\beta) = 9$, $f(\alpha_4) = 7$, $f(\alpha_8) = 4$, $f(\alpha'_k) = 12$,

$f(\alpha_i) = i, i = 1, 2, 3, 5, 6,$

$f(\alpha_i) = i + 1, i = 7, 9, 10.$

From all the above cases, $|e_f(0) - e_f(1)| \leq 1$. Thus G is sum of power n divisor cordial graph.

Example 2.6. A sum of power n divisor cordial labeling of duplication of vertex α_{10}.

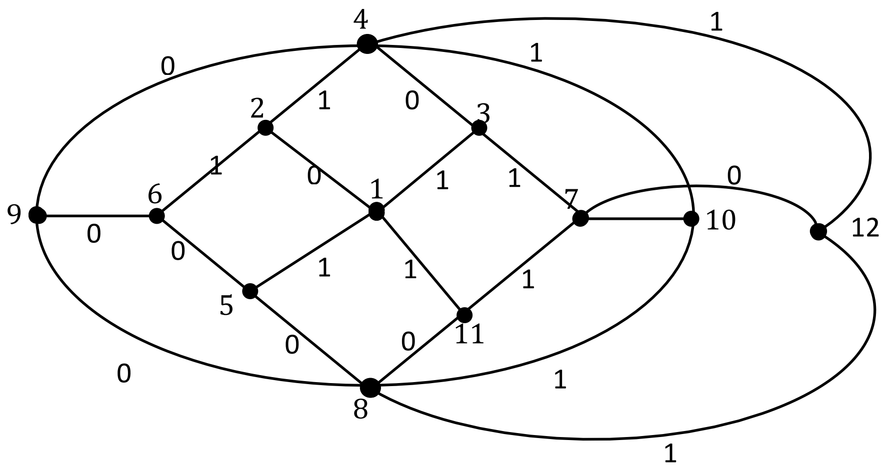

Figure 2.3.

Theorem 2.7. The switching of a central vertex β in the Herschel graph H_s is a sum of power n divisor cordial graph.

Proof. Let H_s be Herschel graph with $|V(H_s)| = 11$ and $|E(H_s)| = 18$. Let β be the central vertex and G be the new graph obtained by switching the central vertex β. Then $|V(G)| = 11$ and $|E(G)| = 20$. Define the vertex labeling $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ as follows;

$f(\beta) = 1,$

$f(\alpha_3) = 11,$

$f(\alpha_8) = 6,$

$f(\alpha_i) = i, i = 4, 5, 9, 10,$

$f(\alpha_i) = i + 1, i = 1, 2, 6, 7.$

From the above labeling pattern, we have $e_f(0) = e_f(1) = 10$.

Hence $|e_f(0) - e_f(1)| \leq 1$. Thus G is sum of power n divisor cordial graph.

Example 2.8. A sum of power n divisor cordial labeling of switching of a central vertex β in H_s.

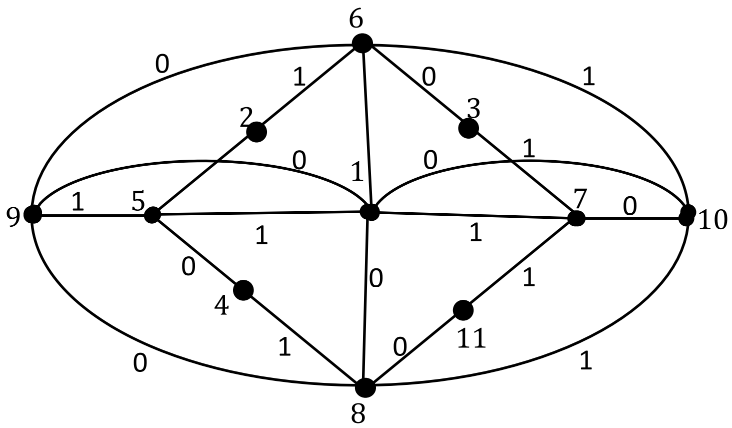

Figure 2.4.

Theorem 2.9. The graph obtained by path union of two copies of Herschel graph H_s is a sum of power n divisor cordial graph.

Proof. Consider two copies of Herschel graph H_s^1 and H_s^2 respectively. Let $V(H_s^1) = \{\alpha, \alpha_i : 1 \le i \le 10\}$ and $V(H_s^2) = \{\beta, \beta_i : 1 \le i \le 10\}$ Then $|V(H_s^1)| = 11$ and $|E(H_s^1)| = 18$ and $|V(H_s^2)| = 11$ and $|E(H_s^2)| = 18$. Let G be the graph obtained by path union of two copies of Herschel graphs H_s^1 and H_s^2. Then $V(G) = V(H_s^1) \cup V(H_s^2)$ and $E(G) = E(H_s^1) \cup E(H_s^2) \cup \{\alpha_8\beta_8\}$. Then $|V(G)| = 22$ and $|E(G)| = 37$. Define the vertex labeling $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ as follows;

Labeling of H_s^1:

$f(\alpha) = 1$, $f(\alpha_3) = 9$, $f(\alpha_5) = 11$, $f(\alpha_8) = 6$, $f(\alpha_9) = 4$, $f(\alpha_{10}) = 10$,
$f(\alpha_i) = i + 1, i = 1, 2, 4, 6, 7.$

Labeling of H_s^2:

$f(\beta) = 12$, $f(\beta_3) = 19$, $f(\beta_5) = 15$, $f(\beta_7) = 20$, $f(\beta_8) = 17$, $f(\beta_9) = 22$,
$f(\beta_i) = i + 12, i = 1, 2, 4, 6.$

From the above labeling pattern, we have $e_f(0) = e_f(1) = 19$.

Hence $|e_f(0) - e_f(1)| \le 1$. Thus G is sum of power n divisor cordial graph.

Example. 2.10. A sum of power n divisor cordial labeling of the path union of H_s^1 and H_s^2.

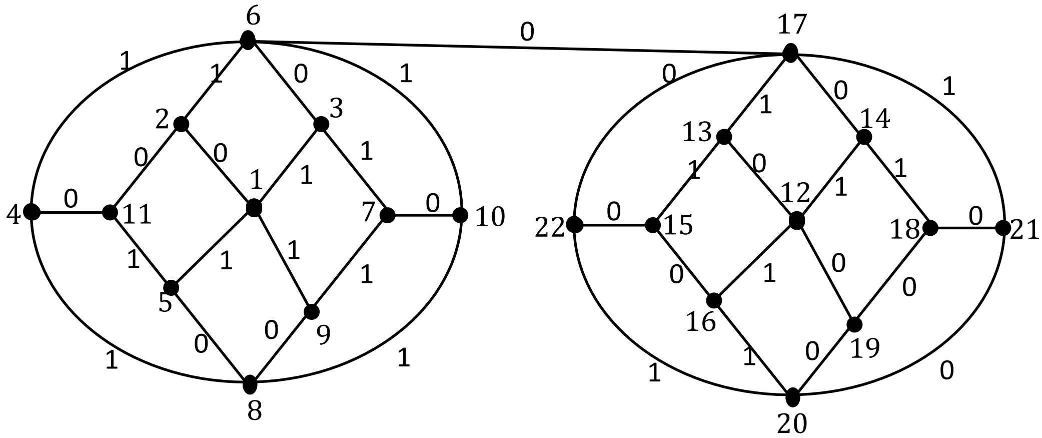

Figure 2.5.

3. Conclusion

In this paper, we have investigated sum of power n divisor cordial graph on special graphs namely Herschel graph H_s, fusion of any two adjacent vertices of degree 3 in Herschel graph H_s, duplication of any vertex of degree 3 in a Herschel graph H_s, switching of a central vertex in the Herschel graph H_s, path union of two copies of Herschel graph H_s are sum of power n divisor cordial graphs.

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